Spring Block 1

Multiplication and division B

Small steps

Small steps

Factor pairs

Notes and guidance

In this small step, children are introduced to factors for the first time. They learn that when they multiply two whole numbers to give a product, both the numbers that they multiplied together are factors of the product. For example, 3 × 5 = 15, so 3 and 5 are factors of 15. 3 and 5 are also referred to as a "factor pair" of 15

They then generalise this further to conclude that a factor of a number is a whole number that divides into it exactly.

Children create arrays using counters to develop their understanding of factor pairs. It is important for children to work systematically when finding the factor pairs of a number in order to ensure that they find all the factors. For example, when finding factor pairs of 12, begin with 1 × 12, then 2 × 6, 3 × 4. At this stage, children should recognise that they have already used 4 in the previous calculation, therefore all factor pairs have been identified.

Things to look out for

- Children may not work systematically, meaning that they could miss some factor pairs.
- Children may find it difficult to understand why not all factors come in pairs, for example 4 × 4 = 16, so this only gives 1 factor of 16, not 2

Key questions

- How can you use arrays to help you find all the factors of a number?
- How do you know that you have found all the factors of ______?
- How do arrays help you to see when a number is not a factor of another number?
- Which number is a factor of every whole number?
- Do factors always come in pairs?
- Do whole numbers always have an even number of factors?

Possible sentence stems

- ______ = ______ × ______, so ______ and ______ are a factor pair of ______
- ______ has ______ factors altogether.

National Curriculum links

- Recognise and use factor pairs and commutativity in mental calculations

Use factor pairs

Notes and guidance

In this small step, children build on their knowledge of factor pairs from the previous step as they use them to write equivalent calculations. For example, as 3 and 4 are a factor pair of 12, this means that 5 × 12 is equivalent to 5 × 3 × 4 or 5 × 4 × 3

Children explore equivalent calculations using different factors pairs, and then practise calculating with them to identify which factor pair produces the easiest calculation to complete mentally. The calculation that is deemed easiest will vary for different children, as they are likely to focus on using the times-tables they are most confident with.

Things to look out for

- Children may need support finding the appropriate factor pairs that will enable them to solve the calculation mentally.
- Children may partition a number rather than finding a factor pair.

Key questions

- How does knowing the factor pairs of 8 help you to find an equivalent calculation to 7 × 8?
- For which number are you going to find the factor pairs?
- Which factor pair is the most helpful to solve the calculation?
- In what order are you going to multiply these numbers?
- Does it matter which factor pair you use?

Possible sentence stems

- The factor pairs of ______ are ______
- 12 = ______ × ______, so ______ × 12 = ______ × ______ × ______
- I can use the factor pairs of ______ to find an equivalent calculation because …

National Curriculum links

- Recognise and use factor pairs and commutativity in mental calculations

Multiply by 10

Notes and guidance

In this small step, children explore multiplying by 10. They need to be able to visualise making a number 10 times the size and understand that "10 times the size" is the same as "multiply by 10".

Children use their understanding that 1 ten is 10 times the size of 1 one and 1 hundred is 10 times the size of 1 ten to support them with this step. A place value chart is useful to show this. They recognise that when multiplying by 10 the digits move one place value column to the left and zero is needed as a placeholder in the now blank column. While children may notice a zero is always used as a placeholder when multiplying a whole number by 10, it is important that they do not develop the misconception that they just add a zero to multiply by 10, as this will cause confusion when multiplying decimals in later learning.

Things to look out for

- Children may move only one digit and misplace the placeholder, for example 45 × 10 = 405
- Children may not realise that calculations of the form 10 × ______ and ______ × 10 can be carried out in the same way.

Key questions

- What do you notice when multiplying by 10?
- What is a placeholder? When do you use placeholders?
- What happens to the digits in a number when you multiply by 10?
- How can you use a place value chart to show multiplying ______ by 10?
- What is ______ multiplied by 10?
- What is 10 lots of ______?

Possible sentence stems

- ______ × 10 = ______
- 10 × ______ = ______
- ______ is 10 times the size of ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)

Multiply by 100

Notes and guidance

Building on the previous step, children learn to multiply whole numbers by 100, understanding that this is the same as multiplying by 10 and then multiplying by 10 again. They need to be able to visualise making a number 100 times the size and understand that "100 times the size" is the same as "multiply by 100".

Children use a place value chart, counters and base 10 to explore what happens to the values of the digits when multiplying by 100. Encourage children to recognise that when multiplying whole numbers by 100, the digits move two place value columns to the left and zeros are needed as placeholders in the now blank columns. As with multiplying by 10 in the previous step, it is important that they do not develop the misconception that they just add two zeros to multiply by 100, as this will cause confusion when multiplying decimals by 100

Things to look out for

- Children may move only some of the digits and misplace the placeholder, for example 45 × 100 = 4,005
- Children may need support to recognise that multiplying by 100 is the same as multiplying by 10 and multiplying by 10 again.

Key questions

- What do you notice when multiplying by 100?
- How can you use multiplying by 10 to help you multiply by 100?
- What happens to the digits when you multiply by 100?
- How can you use a place value chart to show multiplying ______ by 100?
- What is ______ multiplied by 100?
- What is 100 lots of ______?

Possible sentence stems

- ______ × 100 = ______ × 10 × 10 = ______ × 10 = ______
- ______ × 100 = ______, so 100 × ______ = ______
- ______ is 100 times the size of ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)

Divide by 10

Notes and guidance

In this small step, children divide whole numbers by 10, with questions that only have whole number answers. They need to be able to visualise making a number one-tenth the size and understand that "one-tenth the size" is the same as "dividing by 10".

Children use concrete resources and a place value chart to see the link between dividing by 10 and the position of the digits of a number before and after the calculation. They recognise that when dividing by 10, the digits move one place value column to the right. They begin to understand that multiplying by 10 and dividing by 10 are the inverse of each other.

Children may notice that in all the examples they see, they need to "remove the zero" to find the answer. Ensure that they do not generalise this too far and use it as their method, as this will cause issues in later learning when looking at decimals.

Things to look out for

- Children may incorrectly conclude that to divide by 10, they always just remove a zero from the number.
- Children may confuse multiplying and dividing by 10, and move the digits in the wrong direction in a place value chart.

Key questions

- What do you notice when dividing by 10?
- Why does this happen?
- What happens to the digits when you divide by 10?
- How can you use a place value chart to show dividing ______ by 10?
- What is ______ divided by 10?
- What number is one-tenth the size of ______?

Possible sentence stems

- ______ ÷ 10 = ______
- ______ = ______ ÷ 10
- ______ is one-tenth the size of ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)

Divide by 100

Notes and guidance

In this small step, children build on their understanding of dividing by 10 and notice the link between dividing by 10 and dividing by 100. They need to be able to visualise making a number one-hundredth the size and understand that "one-hundredth the size" is the same as "dividing by 100".

Children use concrete resources and a place value chart to see the link between dividing by 100 and the position of the digits before and after the calculation. They realise that when dividing by 100, the digits move two place value columns to the right. They begin to understand that multiplying by 100 and dividing by 100 are the inverses of each other.

Money is a good real-life context for this small step, as exchanging, for example, pounds for pence can be used for the concrete stage.

Things to look out for

- Children may need support in recognising that one-hundredth the size is the same as dividing by 100
- Children may divide by 10 instead of 100
- Children may confuse multiplying and dividing by 100, and move the digits in the wrong direction.

Key questions

- What happens when you divide a number by 10 and then divide the answer by 10 again? How does the final answer compare to the original number?
- How can you use dividing by 10 to help you divide by 100?
- What happens to the digits in a number when you divide by 100?
- How can you use a place value chart to show dividing ______ by 100?
- What is ______ divided by 100?
- What number is one-hundredth the size of ______?

Possible sentence stems

- ______ ÷ 100 = ______ ÷ 10 ÷ 10 = ______ ÷ 10 = ______
- ______ ÷ 100 = ______, so ______ = ______ ÷ 100
- ______ is one-hundredth the size of ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 (Y5)

Related facts – multiplication and division

Notes and guidance

In this small step, children bring together the skills learnt so far in this block as they explore calculations related to known facts.

Children explore scaling facts by 10 and 100, for example using the fact that 4 × 7 = 28 to derive 4 × 70 = 280 and 4 × 700 = 2,800. They then look at this relationship with division, for example using 12 ÷ 3 = 4 to derive 120 ÷ 3 = 40 and 1,200 ÷ 3 = 400. Care should be taken to ensure that children do not also think that 12 ÷ 30 = 40. This is a good opportunity to remind children that multiplication is commutative, but division is not.

A range of representations are used to make the link between multiples of 1, 10 and 100 that will be familiar to children from previous steps in this block and in Year 3

Things to look out for

- Children may derive incorrect division facts by using the rules that they have learnt about related multiplication facts.
- Children may try to find results by calculation rather than recognising the relationship between one fact and another.

Key questions

- What is the same and what is different about the two calculations?
- How can you represent the calculation using place value counters?
- How does knowing that ______ is 10 times the size of ______ help you to complete the calculation?
- What calculation do you know that would help with this one?

Possible sentence stems

- ______ × ______ ones is equal to ______ ones,
  so ______ × ______ tens is equal to ______ tens.
- ______ ÷ ______ is equal to ______,
  so ______ tens ÷ ______ is equal to ______ tens.

National Curriculum links

- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Informal written methods for multiplication

Notes and guidance

In this small step, children use a variety of informal written methods to multiply a 2-digit number by a 1-digit number.

Children follow a clear progression of methods and representations to support their understanding. They begin by using place value charts to recognise multiples of a number and make the link to repeated addition.

The use of base 10 encourages children to partition the tens and ones and unitise the tens, laying the foundations for later work. Part-whole models are used to illustrate the informal method of partitioning. Children use number lines, along with their knowledge of multiplying by 10. For example, to work out 32 × 4 they count along a number line to show 10 × 4 + 10 × 4 + 10 × 4 + 2 × 4. They may also use their knowledge of factor pairs from earlier in the block to multiply.

Things to look out for

- Children may not use the correct place value, multiplying tens as ones, for example 34 × 6 = 3 × 6 + 4 × 6
- Children may conflate the partitioning and factorising methods, for example when calculating 4 × 18, they may do 4 × 9 + 4 × 2

Key questions

- What is the same and what is different about multiplying by 1s and multiplying by 10s?
- How would you explain this method?
- What is the most efficient way to work out ______ × ______?
- How could you use a number line to work out this calculation?
- How could you use a part-whole model to partition into tens and ones?

Possible sentence stems

- ______ partitioned into tens and ones is ______ and ______
- ______ × ______ = ______ tens × ______ + ______ ones × ______ = ______ tens + ______ ones = ______

National Curriculum links

- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects
- Recognise and use factor pairs and commutativity in mental calculations

Multiply a 2-digit number by a 1-digit number

Notes and guidance

In this small step, children progress from multiplying using informal written methods to the formal written method. The short multiplication method is introduced for the first time, initially in an expanded form and then in the formal short single-line form.

Children first do calculations where there are no exchanges, then move on to one and two exchanges. Place value counters in place value charts are used to illustrate the structure of the short multiplication by presenting the concrete model alongside the formal written method.

Concrete manipulatives alongside abstract calculations are particularly useful to support children's understanding of exchanges.

Things to look out for

- Children may exchange ones or tens incorrectly, often by missing zeros or including zeros erroneously.
- Children may not include digits created through exchanging, either by not writing them down when completing the exchange or neglecting to include them in the calculation afterwards.
- When exchanges are performed, if digits are written in the incorrect place, this can lead to errors with the rest of the calculation.

Key questions

- What is the same and what is different about multiplying by 1s and multiplying by 10s?
- How does the written method match the representation?
- Which column should you start with?
- What is the same and what is different about the different methods?

Possible sentence stems

- ______ ones × ______ = ______ ones,
  ______ tens × ______ = ______ tens
- To multiply a 2-digit number by ______, you multiply the ______ by ______ and the ______ by ______
- ______ tens multiplied by ______ plus the ten I exchange is equal to ______ tens.

National Curriculum links

- Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout

Multiply a 3-digit number by a 1-digit number

Notes and guidance

Following on from the previous step, children extend the formal written method to multiplying a 3-digit number by a 1-digit number. They continue to use the short multiplication method, but now with more columns. Children need to be secure with the previous step before moving on to this one.

Place value counters in place value charts are again used to model the structure of the formal method, allowing children to gain a greater understanding of the procedure, particularly where exchanges are needed. They continue to use the counters to exchange groups of 10 ones for 1 ten and also exchange 10 tens for 1 hundred and 10 hundreds for 1 thousand. This is mirrored by the positioning of the exchanged digit in the formal written method.

The focus here is on the short written method, but the expanded method could be used to support understanding for children who need it.

Things to look out for

- The use of a zero in the ones or tens column can sometimes expose misunderstandings, as children can be unsure of multiplying by zero.
- Children may omit the exchange or include the exchange in an incorrect place on the formal written method.

Key questions

- How could you use counters to represent the multiplication?
- How does the written method match the representation?
- Which column should you start with?
- Do you need to make an exchange? What exchange can you make?
- What is the same and what is different about multiplying a 3-digit number by a 1-digit number and multiplying a 2-digit number by a 1-digit number?

Possible sentence stems

- ______ ones × ______ = ______ ones
  ______ tens × ______ = ______ tens
  ______ hundreds × ______ = ______ hundreds
- ______ tens/hundreds multiplied by ______ plus the ten/hundred from the exchange is equal to ______

National Curriculum links

- Multiply 2-digit and 3-digit numbers by a 1-digit number using formal written layout

Divide a 2-digit number by a 1-digit number (1)

Notes and guidance

In this small step, children use their division facts from the Autumn term to build on their knowledge of dividing a 2-digit number by a 1-digit number from Year 3

Initially, children carry out divisions where the tens and ones are both divisible by the number being divided by without any remainders, for example 96 ÷ 3 and 84 ÷ 4. They then move on to calculations where they need to exchange between tens and ones, for example 96 ÷ 4. Place value counters are used to explore the sharing structure of division. Children do not need to use the formal short division method at this stage and may use informal jottings or representations such as a part-whole model to record their working instead.

Things to look out for

- Children may partition the 2-digit number correctly, but then divide the tens as if they are ones, for example 96 ÷ 3 = 9 ÷ 3 + 6 ÷ 3
- Instead of using their times-tables knowledge, children may revert to less efficient methods such as drawing circles, then drawing dots to share between the circles.
- Children may always partition into tens and ones when other forms of partitioning are more appropriate.

Key questions

- How do you partition a 2-digit number into tens and ones? How else can you partition a 2-digit number?
- Which is the most efficient way to partition the number so you can divide both parts by ______?
- If you cannot share all of the tens equally, what do you need to do?
- How can you represent the division using a part-whole model?

Possible sentence stems

- ______ tens divided by ______ = ______ tens each
- ______ ones divided by ______ = ______ ones each
- I cannot share all of the tens equally, so I need to …

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Divide a 2-digit number by a 1-digit number (2)

Notes and guidance

In this small step, children continue to explore dividing a 2-digit number by a 1-digit number, but in this step the focus is on calculations with remainders.

Children enountered remainders in Year 3, so this concept is not new but it may need reinforcing.

Using place value counters to illustrate the sharing structure of division helps children to see what is meant by the remainder. Such representations should highlight the fact that the remainder can never be greater than the number they are dividing by.

Things to look out for

- Children may not fully divide and so will have a remainder that is greater than the number they are dividing by.
- Children may partition the 2-digit number correctly, but then divide the tens as if they are ones, for example $95 \div 3 = 9 \div 3 + 5 \div 3$
- Children may revert to less efficient methods, such as drawing circles and then drawing dots to share between the circles.
- Children may divide the whole number rather than partitioning into tens and ones and then unitising the tens.

Key questions

- Can the counters be shared equally? If not, how many are left over?
- What does "remainder" mean?
- What is the greatest remainder you can have when you are dividing by ______?
- How can you partition a 2-digit number?
- If you cannot share all the tens equally, what do you need to do?
- If you cannot share all the ones equally, what happens?
- How do you know that $43 \div 2$ will have a remainder?

Possible sentence stems

- If I am dividing by ______, then the greatest possible remainder is ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Divide a 3-digit number by a 1-digit number

Notes and guidance

In this small step, children continue to develop their understanding of division by extending from dividing 2-digit numbers in the previous two steps to dividing 3-digit numbers.

Place value counters are again used to represent the calculations, so that children can make sense of exchanges that are needed to complete the division.

Part-whole models are also used to show how flexible partitioning can support the process of division by looking for multiples of the number being divided by.

The step starts with divisions that do not leave a remainder, before progressing to divisions with remainders.

By the end of this step, children should have a good understanding of division that will support them when they move on to the formal written method in Year 5

Things to look out for

- Children may partition the 3-digit number correctly, but then divide the hundreds and tens as if they are ones, for example $846 \div 2 = 8 \div 2 + 4 \div 2 + 6 \div 2$
- Children may divide the whole number rather than partitioning into hundreds, tens and ones and then unitising the hundreds and tens.

Key questions

- How do you partition a 3-digit number into hundreds, tens and ones?
- How else can you partition a 3-digit number?
- What is the best way to partition the number to help you work out the division?
- If you cannot share all of the hundreds/tens equally, what do you need to do?
- How can you represent the division using a part-whole model?

Possible sentence stems

- ______ hundreds divided by ______ = ______ hundreds
- ______ tens divided by ______ = ______ tens
- ______ ones divided by ______ = ______ ones
- There is ______ left over, so I need to exchange it for ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers

Correspondence problems

Notes and guidance

In this small step, children consolidate their understanding of correspondence problems from Year 3, using multiplication to work out the number of possible combinations of sets of items.

Children use a range of representations and contexts to support them. Using tables helps to encourage children to adopt a systematic approach to finding all of the possible combinations in a given context. Children then generalise to make the link between the number of possibilities for each item and using multiplication to find the total number of combinations.

Once confident with finding all possible combinations for two sets of items children may begin to explore finding all possible combinations for three sets of items.

Things to look out for

- Children may see the same choices in a different order as a different choice.
- Children may need support to work systematically when listing all possibilities.
- Children may add instead of multiply the number of possibilities for each item.

Key questions

- How can you use a table to help you find the possible combinations?
- How can you be sure that you have listed all the possibilities?
- How could you use a code to help you list the combinations?
- What do you notice about the number of choices for each item and the total number of combinations?
- How can you check your answer?
- Does the order in which you make your choices matter?

Possible sentence stems

- For every ______, there are ______ ______
- Altogether, there are ______ × ______ = ______ possible combinations.

National Curriculum links

- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Efficient multiplication

Notes and guidance

In this small step, children consolidate their knowledge and understanding of multiplication and begin to make decisions regarding the most efficient or appropriate methods to use in a range of contexts.

Children look at times-tables facts, building strategies for finding unknown facts that will support them to strengthen their fluency of times-tables. They then examine a range of strategies for multiplying a 2-digit number by a 1-digit number. Finally, they use arrays to explore multiplicative structure, in particular the associative law and distributive law.

Things to look out for

- Children may conflate different methods, leading to misunderstanding.
- Children may partition the numbers correctly, but then multiply the tens as if they are ones, for example $34 \times 6 = 3 \times 6 + 4 \times 6$
- Children may attempt to learn the different methods procedurally. It is vital that children understand how they are manipulating the numbers, rather than try to remember a long series of instructions.

Key questions

- Which method do you find most efficient? Explain how this method works.
- What is the most efficient way to work out ______ × ______?
- What happens if you double one factor and halve the other?
- How could you use factor pairs to help you calculate?

Possible sentence stems

- ______ × ______ = ______ × ______ + ______ × ______
- ______ × ______ = ______ × ______ – ______ × ______
- ______ × ______ = ______ × ______ × 2
- ______ × ______ = ______ × ______ ÷ 2

National Curriculum links

- Solve problems involving multiplying and adding, including using the distributive law to multiply 2-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects