Multiples of 3

Notes and guidance

This small step revisits learning from Year 3 around multiplying by 3 and the 3 times-table.

Children explore the link between counting in 3s and the 3 times-table to understand multiples of 3 in a range of contexts. They use familiar representations such as number tracks and hundred squares to represent multiples of 3. They explore how to recognise if a number is a multiple of 3 by finding its digit sum: if the sum of the digits of a number is a multiple of 3, then the number itself is also a multiple of 3

This small step includes multiples of 3 up to 3 × 12 and will be useful support for learning multiples of 6 and 9 in future steps.

Things to look out for

- Children may think that any number with 3 ones is a multiple of 3
- An early mistake when counting in 3s will affect all subsequent multiples.
- Children may always begin counting from 3 to find a larger multiple of 3, when they could use the multiples they already know to find the new information.

Key questions

- What is the next multiple of 3?
- What is the multiple of 3 before ______?
- How many 3s are there in ______?
- How do you find the digit sum of a number?
- How can you tell if a number is a multiple of 3?
- Are the multiples of 3 odd or even?

Possible sentence stems

- The next multiple of 3 is ______
- The multiple of 3 before ______ is ______
- I know ______ is a multiple of 3 because …

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

Multiply and divide by 6

Notes and guidance

In this small step, children build on their knowledge of the 3 times-table to explore the 6 times-table. The step aims to embed the children's fluency skills with the 6 times-table, while also providing them with strategies to use the multiplication facts they know to find unknown facts.

Children explore the fact that the 6 times-table is double the 3 times-table. Children who are confident in their times-tables can also explore the link between the 5 and 6 times-tables. They use the fact that multiplication is commutative to derive values for the 6 times-tables. This is developed further with division facts, where children explore fact families to embed their understanding of division as the inverse of multiplication.

Things to look out for

- Children may always start at 1 × 6 = 6 and recite the times-table, rather than use the number facts they know to find the facts they are not as secure with.
- When writing fact families, children may follow the pattern from multiplication and see division as commutative, for example writing 42 ÷ 6 = 7 so 6 ÷ 42 = 7
- Children may not recognise that when they are dividing by a greater number they get a smaller answer.

Key questions

- How many equal groups do you have?
  How many are there in each group?
  How many are there altogether?
- What does each number in the calculation represent?
- What does commutative mean?
- Is multiplication/division commutative?
- How can you use facts from the 3 times-table to work out facts from the 6 times-table?

Possible sentence stems

- 6 lots of ______ is ______
- ______ shared into 6 equal groups is ______
- Multiplying by 6 is the same as multiplying by ______ twice.
- ______ × 6 = double ______ × 3

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

6 times-table and division facts

Notes and guidance

Building on the previous step, children use known facts to become more fluent in using the 6 times-table.

As in the previous step, they apply knowledge of the 3 times-table and understand that each multiple of 6 is double the corresponding multiple of 3

Children use their knowledge of other times-tables to find values for the 6 times-table, for example finding that $6 \times 7 = 42$ because $5 \times 7 = 35$ and $1 \times 7 = 7$, so $35 + 7 = 42$

It is important that children practise the related division facts as well as the multiplication facts associated with the 6 times-table. Fluency with the 6 times-table will also help children to work out the 12 times-table in future steps.

Things to look out for

- Children may confuse different terminology to describe multiplication and division such as "equal groups", "lots of", "times", "multiple" and so on.
- An early mistake when counting in 6s will affect all subsequent multiples.
- Children may not see the link between 6 × ______ and other multiples such as 5 × ______ and 1 × ______

Key questions

- How can you use facts from the 3 times-table to work out facts in the 6 times-table?
- How can you use facts from the 5 times-table to work out facts in the 6 times-table?
- If you know a multiplication sentence, what division sentences can you find?
- What is the fact family for the calculation?

Possible sentence stems

- 6 multiplied by ______ is equal to ______
- ______ × 6 = double ______ × 3
- ______ × 6 = ______ × 5 + ______
- ______ × 6 = ______, so ______ ÷ 6 = ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

Multiply and divide by 9

Notes and guidance

In this small step, children are introduced to the 9 times-table. They use a range of strategies to support their fluency, such as looking for number patterns and finding unknown number facts from known facts, for example subtracting from the 10 times-table or tripling the 3 times-table, and these will be built upon later in the block.

Children explore the structure of the 9 times-table using a range of models and pictorial representations, and by exploring multiples of 9 in context. They also use commutativity with the facts they already know from other times-tables.

Children find division facts and explore fact families to embed their understanding of division as the inverse of multiplication.

Things to look out for

- When finding multiplication facts, children may always start at 1 × 9 = 9 and recite the times-table rather than using the number facts they know to find the facts they are not as secure with.
- When writing fact families, children may follow the pattern from multiplication and see division as commutative, writing examples such as 54 ÷ 9 = 6 so 9 ÷ 54 = 6

Key questions

- How many equal groups are there?
  How many are there in each group?
  How many are there altogether?
- How can you use the 10 times-table to work out the 9 times-table?
- How can you use the 3 times-table to work out the 9 times-table?
- What does each number in the calculation represent?
- What patterns can you see in the 9 times-table?

Possible sentence stems

- 9 lots of ______ is equal to ______
- ______ groups of ______ is equal to ______ groups of ______
- ______ × 10 = ______, so ______ × 9 = ______ – ______ = ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

9 times-table and division facts

Notes and guidance

Building on the previous step, children become more fluent using the 9 times-table and apply the multiplication and division facts in a wide variety of contexts.

To establish the facts, children use strategies such as using the 10 times-table to derive the 9 times-table, and understanding that each multiple of 9 is triple the equivalent multiple of 3

They investigate finding the digit sum and look for patterns that will support them in identifying multiples of 9: if the sum of the digits of a number is a multiple of 9, then the number itself is also a multiple of 9. This, and the corresponding rule for the 3 times-table, will support their learning in the next step, where they compare the 3, 6 and 9 times-tables.

Things to look out for

- Children may confuse different terminology to describe multiplication and division, such as “equal groups”, “lots of”, “times”, “multiple” and so on.
- An early mistake when counting in 9s will affect all subsequent multiples.
- Children may use tricks to find multiplication facts in the 9 times-table but not be able to use these to find the related division facts.

Key questions

- How could you use the 10 times-table to work out the 9 times-table?
- If you know a multiplication sentence, what division sentences can you find?
- How can you tell if a number is a multiple of 9?
- How can you use the 3 times-table to work out facts in the 9 times-table?

Possible sentence stems

- ______ × 9 = ______ × 9 + ______ × 9
- ______ × 9 = ______ – ______ = ______
- ______ × 9 = ______, so ______ ÷ 9 = ______
- Multiplying by 9 is the same as multiplying by ______ and then multiplying by ______ again.

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

The 3, 6 and 9 times-tables

Notes and guidance

In this small step, children make links between the 3, 6 and 9 times-tables to deepen their understanding and embed fluency with these times-tables.

This is done by exploring the structure of the times-tables using resources such as arrays and hundred squares, as well as via tasks that require children to reason and explore number facts to look for structural patterns.

On completion of this step, children should be confident with their 2, 3, 4, 5, 6, 8, 9 and 10 times-tables before moving on to look at the remaining times-tables later in the block.

Things to look out for

- Children may see the pattern of doubling 3 times-table facts to find 6 times-table facts, then make the mistake of assuming that they can double the 6 times-table facts to find 9 times-table facts.
- Children may rely on reciting the times-tables, rather than using known facts at other points in the times-tables to help them.
- Even when children are secure in multiplication facts, they may not be confident with the corresponding divisions.

Key questions

- What links can you see between the 3 and 6 times-tables?
- What links can you see between the 3 and 9 times-tables?
- What other times-tables can you use to help find the multiplication facts?
- If you know one multiplication fact, what other multiplication fact do you know? What division facts do you know?
- How do you know if a number is a multiple of 3/6/9?

Possible sentence stems

- Double ______ × 3 = ______ × 6
- Triple ______ × 3 = ______ × 9
- 3 lots of ______ and 6 lots of ______ = 9 lots of ______
- ______ × 3 × 3 = ______ × ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

Multiply and divide by 7

Notes and guidance

In this small step, children use their knowledge of multiples and count in 7s to make the link between repeated addition and multiplication.

Children apply their knowledge of equal groups and use a range of concrete and pictorial representations to deepen their understanding of multiplying by 7. They also draw on ideas from previous steps to explore flexible partitioning to show, for example, $8 \times 7 = 5 \times 7 + 3 \times 7$ or $8 \times 7 = 8 \times 5 + 8 \times 2$

Children also explore dividing by 7 through sharing into 7 equal groups and grouping into 7s.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones they do not know as well.
- Children may not be able to identify which number in a number sentence corresponds with which number in a context.
- Children may find all multiplication facts by starting from 1×7 and then reciting their times-table facts, rather than using facts they know to find the facts they do not know.

Key questions

- How many equal groups are there?
- How many lots of 7 do you have?
- How many groups of 7 are there in ______?
- What can you partition ______ into to help you multiply ______ by 7?
- If you know this, what else do you know?
- How can you use the 5/6/8 times-table to find a fact in the 7 times-table?

Possible sentence stems

- ______ × 7 = ______ × 7 + ______ × 7
- ______ × 7 = ______ × 8 – ______ = ______
- There are 7 groups of ______ in ______

National Curriculum links

- Count in multiples of 6, 7, 9, 25 and 1,000
- Recall multiplication and division facts for multiplication tables up to 12 × 12

7 times-table and division facts

Notes and guidance

In this small step, children bring together their knowledge of multiplying and dividing by 7 in order to become more fluent in the 7 times-table.

Children construct fact families and use concrete and pictorial representations to make links between multiplication and division. It is important that children understand the structure of the multiplication table and can derive unknown facts from known facts. Children explore links between multiplication tables, investigating how this can help with mental strategies for calculation, such as $9 \times 7 = 9 \times 5 + 9 \times 2$. This step could also be an opportunity to use the 6 and 8 times-tables to derive the 7 times-table, for example $9 \times 7 = 9 \times 8 - 9$ or $9 \times 7 = 9 \times 6 + 9$. Drawing arrays is a useful way of helping children to see these links.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones that they do not know as well.
- Children may find all multiplication facts by starting from 1×7 and then reciting their times-table facts, rather than using facts they know to find the facts they do not know.

Key questions

- How many lots of 7 do you have?
- What is the same and what is different about the number facts?
- How does the 7 times-table help you work out the answers?
- What strategies can you use to work out a 7 times-table fact that you do not yet know? What other times-tables can you use?

Possible sentence stems

- ______ × 7 = ______ × 5 + ______ × 2
- ______ × 7 = ______ × 8 – ______
- ______ × 7 = ______ × 6 + ______
- There are 7 groups of ______ in ______
- There are ______ groups of 7 in ______

National Curriculum links

- Count in multiples of 6, 7, 9, 25 and 1,000
- Recall multiplication and division facts for multiplication tables up to 12 × 12

11 times-table and division facts

Notes and guidance

In this small step, children build on their knowledge of the 1 and 10 times-tables to explore the 11 times-table. They recognise that they can partition 11 into 10 and 1 and use known facts to support their understanding, for example
$7 \times 11 = 7 \times 10 + 7 \times 1 = 77$

They use a range of concrete and pictorial representations to deepen their understanding of multiplying by 11 and to make links between multiplying and dividing by 11. They explore dividing by 11 through sharing into 11 equal groups and grouping into 11s.

At this stage, children should already know the majority of facts from other times-tables, so highlighting the importance of commutativity is key in this step.

Things to look out for

- Children may need support to use the multiplication facts that they are confident in to find the ones that they do not know as well.
- Children may not realise that 110, 121, 132 and so on are multiples of 11, as the previous multiples of 11 all have repeated digits, for example 66, 77, 88

Key questions

- How many equal groups are there?
- How many lots of 11 do you have?
- How many groups of 11 are there in ______?
- What can you partition 11 into to help you?
- How can you use base 10 to work out ______ × 11?
- How can you use place value counters to work out ______ ÷ 11?
- How can you show this using an array?

Possible sentence stems

- ______ × 11 = ______
- ______ × 11 = ______ × 10 + ______ × 1
- There are 11 groups of ______ in ______
- There are ______ groups of 11 in ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Recognise and use factor pairs and commutativity in mental calculations

12 times-table and division facts

Notes and guidance

In this small step, children build on their knowledge of the 2 and 10 times-tables to explore the 12 times-table. They recognise that they can partition 12 into 10 and 2 and use known facts to support their understanding, for example $7 \times 12 = 7 \times 10 + 7 \times 2 = 84$. They also build on their knowledge of the 6 times-table, recognising that multiplying by 12 is the same as multiplying by 6 and then doubling.

Children use a range of concrete and pictorial representations to deepen their understanding of multiplying by 12 and to make links between multiplying and dividing by 12. They explore dividing by 12 through sharing into 12 equal groups and grouping into 12s.

At this stage, children should already know multiplication facts from other times-tables, so highlighting the importance of commutativity is key in this step.

Things to look out for

- Children may need support to use known multiplication facts to find new ones.
- Children may find all multiplication facts by starting from 1×12 and then reciting their times-table facts, rather than using facts that they know.

Key questions

- How many equal groups are there?
- How many lots of 12 do you have?
- How many groups of 12 are there in ______?
- What can you partition 12 into to help you?
- How can you use base 10 to work out ______ × 12?
- How can you use place value counters to work out ______ ÷ 12?

Possible sentence stems

- ______ × 12 = ______ × 10 + ______ × 2
- ______ × 12 = double ______ × 6
- There are 12 groups of ______ in ______
- There are ______ groups of 12 in ______

National Curriculum links

- Recall multiplication and division facts for multiplication tables up to 12×12
- Recognise and use factor pairs and commutativity in mental calculations

White Rose Maths

Multiply by 1 and 0

Notes and guidance

In this small step, children explore the effect of multiplying by 1. They notice that when they multiply a number by 1, the result will always be the number itself.

This small step also focuses on multiplying by zero. Children learn that when multiplying any number by zero the result is always zero.

A common misconception with this small step is that children confuse the result of multiplying by zero with multiplying by 1. Ensure pictorial representations are used to address this misconception, so that children can see that 4 × 0 is the same as 4 lots of zero, which is equal to zero.

Things to look out for

- Children may use addition instead of multiplication, for example 1 × 1 = 2 and 8 × 1 = 9
- Children may confuse the result of multiplying by zero with multiplying by 1
- When working out a longer multiplication, for example 3 × 4 × 5 × 0, children may start working from left to right rather than realising that as they are mutiplying by zero the answer must be zero.

Key questions

- What does "zero" mean? How can you multiply by zero?
- What do you notice about the results of multiplying numbers by zero?
- What does "multiplying by 1" mean?
- What do you notice about the results of multiplying numbers by 1?
- What is the same and what is different about multiplying by 1 and multiplying by zero?

Possible sentence stems

- Any number multiplied by zero is equal to ______
- Any number multiplied by 1 is equal to ______
- ______ groups of one = ______
- ______ groups of zero = ______

National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

Divide a number by 1 and itself

Notes and guidance

In this small step, children apply their knowledge of division and explore what happens to a number when they divide it by 1 or itself.

Children can sometimes confuse the result of dividing a number by 1 with dividing a number by itself. Ensure concrete and pictorial representations are used to address this misconception, including examples that involve both structures of division. Stem sentences can be used to encourage children to see this, for example: 5 grouped into 5s is equal to 1 ($5 \div 5 = 1$) and 5 grouped into 1s is equal to 5 ($5 \div 1 = 5$).

Following on from the previous small step, children may try to divide a number by zero and it should be highlighted that this is not possible.

Things to look out for

- Children may assume that division is commutative and think that $12 \div 1 = 1 \div 12$
- Children may confuse the result of dividing a number by 1 with dividing the number by itself.
- Children may think a number divided by itself is zero.

Key questions

- How many equal groups of ______ can you make?
- What is ______ shared equally into 1 group?
- What is ______ grouped into groups of 1?
- What is the same and what is different about multiplying by 1 and dividing by 1?
- What is the same and what is different about dividing a number by 1 and dividing a number by itself?

Possible sentence stems

- When you divide a number by itself, the answer is …
- When you divide a number by ______, the number remains the same.
- There are ______ 1s in ______
- There is 1 ______ in ______

National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

White Rose Maths

Multiply three numbers

Notes and guidance

In this small step, children apply their knowledge of multiplication to multiply three numbers together.

They are introduced to the idea of the associative law (but do not need to know it by name), which focuses on the fact that it does not matter how they group the numbers when they multiply. For example, 4 × 5 × 2 = (4 × 5) × 2 = 20 × 2 = 40 or 4 × (5 × 2) = 4 × 10 = 40

Encourage children to link this idea to commutativity and change the order of the numbers to group them more efficiently.

Counters and cubes are effective concrete resources to use during this step to support children's understanding of the associative law.

Things to look out for

- Children may need support ordering the numbers to group them more efficiently.
- If children are not confident with their times-table facts, they may struggle with multiplying three numbers.
- Children may automatically work from left to right without looking at the most efficient way to complete a calculation.

Key questions

- Do you have to multiply the numbers from left to right?
- Which pair(s) of numbers do you know the product of?
- How will you decide which order to do the multiplication in?
- What is the same about these calculations/arrays?
- Which order do you find easier to calculate efficiently?
- If you worked out the calculation in a different order, would you get a different answer? Why/why not?

Possible sentence stems

- I am going to work out ______ × ______ first, because …
- To work out ______ × ______ × ______, I can first calculate ______ × ______ and then multiply the answer by ______
- If ______ × ______ = ______, then ______ × ______ × ______ = ______

National Curriculum links

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers